Finance

$$FV = PV \cdot \left(1 + \frac{r}{100 \cdot n}\right)^{n \cdot t}$$

yıllık

$n \cdot t \to$ yıl

dönem

4, 12

(P2)

1a. *[3 marks]*

On 1st January 2020, Laurie invests $P in an account that pays a nominal annual interest rate of 5.5 % compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, *r*.

Find the value of *r*, giving your answer to four significant figures.

1b. *[3 marks]*

Laurie makes no further deposits to or withdrawals from the account.

Find the year in which the amount of money in Laurie's account will become double the amount she invested.

2020

12

$\boxed{2032}$

$$FV = P \cdot \boxed{\left(1 + \frac{5.5}{100 \times 4}\right)^4}$$

$1.0561448 \sim \boxed{1.056}$

1b) $\cancel{P} \cdot \left(1 + \frac{5.5}{100 \times 4}\right)^{4t \to x} = 2\cancel{P}$

solve

$\boxed{\text{graph}}$

$$\left(1 + \frac{5.5}{100 \times 4}\right)^{4 \cdot x} = 2$$

-2

$$\log_{\left(1 + \frac{5.5}{400}\right)} 2 = 4x$$

$x = 12.68 \sim 12.7$

(P2)

$a_n = a_1 + (n-1)\,2400$

$\Rightarrow \boxed{a_n = 2400n + 67600}$

3a. *[2 marks]*

Helen and Jane both commence new jobs each starting on an annual salary of \$70,000.
At the start of each new year, Helen receives an annual salary increase of $\boxed{\$2400}$.

Let $\$H_n$ represent Helen's annual salary at the start of her nth year of employment.

Show that $H_n = 2400n + 67\,600$. ✓

3b. *[1 mark]*

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

geometric

0.03

1+0.03

$a_n = a_1 \times r^{n-1}$

$= \boxed{70{,}000 \times (1.03)^{n-1}}$

Jane's annual salary, $\$J_n$, at the start of her nth year of employment is given by $J_n = 70\,000(1.03)^{n-1}$.

Given that J_n follows a geometric sequence, state the value of the common ratio, r. ✓

3c. *[3 marks]*

At the start of year N, Jane's annual salary exceeds Helen's annual salary for the first time.

Find the value of N.

3d. *[2 marks]*

For the value of N found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar.

3e. *[4 marks]*

Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar.

9

$70{,}000 \times (1.03)^{n-1}$

$a_1 \times \dfrac{1-r^n}{1-r}$

$\Downarrow$

$70{,}000 \times \dfrac{1-(1.03)^9}{1-1.03} = \boxed{\$711{,}137}$

(P2)

4a. *[2 marks]*

$\frac{7}{12}$

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

Find the first term of the sequence, u_1.

$\frac{2}{3} \cdot \frac{7}{8} = \frac{7}{12}$

4b. *[3 marks]*

Find S_∞.

4c. *[4 marks]*

Find the least value of n such that $S_\infty - S_n < 0.001$.

4b) $\frac{a_1}{1-r} = \frac{\frac{7}{12}}{1-\frac{7}{8}} = \frac{14}{3}$

4c) $S_\infty - S_n < 0.001$

$$\frac{14}{3} - \left[\frac{7}{12} \cdot \frac{1-\left(\frac{7}{8}\right)^n}{1-\frac{7}{8}}\right] < 0.001$$

$x = 63.2695$

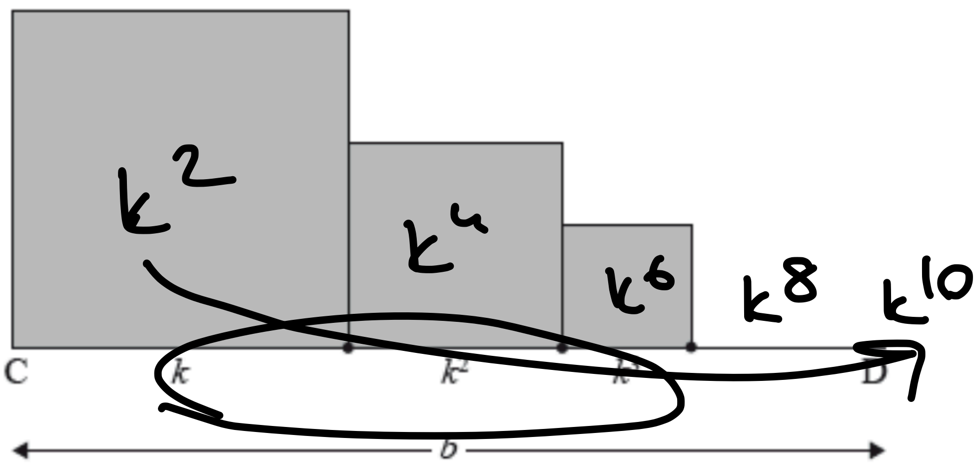
$\boxed{n = 64}$

(P1)

9. *[9 marks]*

The following diagram shows [CD], with length b cm, where $b > 1$. Squares with side lengths k cm, k^2 cm, k^3 cm, …, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.

diagram not to scale



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.

$$\frac{a_1}{1-r} \Rightarrow \frac{k^2}{1-k^2} = \frac{9}{16}$$

$$16k^2 = 9 - 9k^2$$

$$25k^2 = 9$$

$$k^2 = \frac{9}{25}$$

$$\boxed{k = \frac{3}{5}}$$

k, k^2, k^3

$\rightarrow r = k$

$$\frac{a_1}{1-r} = \frac{k}{1-k}$$

$$\frac{\frac{3}{5}}{1-\frac{3}{5}} = \frac{3}{2}$$

(P1)

13a. *[1 mark]*

Jashanti is saving money to buy a car. The price of the car, in US Dollars (USD), can be modelled by the equation

$$\boxed{P = 8500\,(0.95)^t.}$$

Jashanti's savings, in USD, can be modelled by the equation

$$\boxed{S = 400t + 2000.} \Rightarrow$$

In both equations t is the time in months since Jashanti started saving for the car.

Write down the amount of money Jashanti saves per month. 400$

13b. *[2 marks]*

TI

Use your graphic display calculator to find how long it will take for Jashanti to have saved enough money to buy the car.

13c. *[3 marks]*

Jashanti does not want to wait too long and wants to buy the car two months after she started saving. She decides to ask her parents for the extra money that she needs.

Calculate how much extra money Jashanti needs.

b) $400t + 2000 > 8500(0.95)^t$

$400t + 2000 - 8500(0.95)^t > 0$

$x = 8.641$

$t = 9$

c) $8500(0.95)^2 - (400 \times 2 + 2000) = \$4871.3$

[Maximum mark: 8]

It is known that the number of trees in a small forest will decrease by 5% each year unless some new trees are planted. At the end of each year, 600 new trees are planted to the forest. At the start of 2021, there are 8200 trees in the forest.

(a) Show that there will be roughly 9060 trees in the forest at the start of 2026. 5

(b) Find the approximate number of trees in the forest at the start of 2041.

a) $\rightarrow 8200(0.95)^{1} + 600$

$\rightarrow (8200(0.95) + 600)\,0.95 + 600$

$8200(0.95)^2 + 600(0.95) + 600$

$8200(0.95)^5 + 600(0.95)^4 + 600(0.95)^3 \ldots$

$600[(0.95)^4 + (0.95)^3 + (0.95)^2 + (0.95)^1 + 1]$

$$600 \cdot \frac{1-(0.95)^5}{1-0.95} = \boxed{2714.62875}$$

9059.632

$\approx \boxed{9060}$

$$600 \cdot \frac{(1-(0.95)^{20})}{1-0.95} + 8200 \cdot (0.95)^{20}$$

$$= 10637.7734\ldots$$

$$\approx \boxed{10638}$$